Jens Høyrup

State formation, administration, "justice" and mathematics in Mesopotamia - and how the interplay developed.

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State and mathematics

The functioning of the modern state presupposes a variety of mathematical technologies – accounting, statistics, and much more.

Mathematics, on its part, needs the institutions of the state (schools, universities, research institutions, etc.) to secure financing, recruitment and the rearing of competence.

At a given moment, the state as well as mathematics largely take the partner "as it is", and none of them appears to the immediate view to depend for its essence on the other.

Actually, the situation of the latest four to six decades or so may be different:

without information technology, the immense increase of administrative control of citizens (to mention but that) would never have been possible.

> And here, please, observe that I am not speakingabout China – you may not know, but control is much more intensive where I come from than here.

This, though not my topic, is an intended background for what I deal with here.

Situations at the emergence of the state as a type of social organization differ:

Most statal systems originated in complex processes

– either as “pristine states” via expanding chiefdoms
– or as “secondary states”, in interaction with (often, indeed, as military protection against) existing states.

As a rule, the involvement of anything than can be considered as mathematics in such processes has been peripheral, if not totally absent.

In a few exceptional cases, however, mathematical technologies appear to have played a major role in the shaping of the state, becoming themselves more sophisticated in the process, developing into recognizable *mathematics*.

– the bureaucratic Inca state and its accounting. I suspect, however, that the general knowledge of the topic is insufficient to trace the connections between the development of the state and that of the *quipu* system.

– Possibly, the (ideological and perhaps less pivotal?) relation between the Maya states and their "chrono-theology"; but the Maya state formation was not pristine.

– The most indisputable case, best (though indirectly) reflected in sources: the formation of states in southern Mesopotamia in the late 4th millennium BCE.

Prolegomena

What do I mean (here) by “mathematics”, and what by “state”?

First mathematics:

The transition to “recognizable mathematics” may be characterized as

> the point where *pre-existent but previously independent* mathematical practices are coordinated through a minimum of at least intuitive understanding of formal relations.

Political anthropologists have discussed the emergence of *statal organization* of society in different terms.

One classic is Morton Fried's *The Evolution of Political Society*. The state arises as

> a collection of specialized institutions and agencies, some formal and others informal, that maintain an order of stratification,

"stratified society" being a society

> in which members of the same sex and equivalent age status do not have equal access to the basic resources that sustain life.

China under the Shang dynasty was thus certainl a stratisfied society – but it was certainly not the beginning, recent and less recent archaeology shows stratification at least 1500 years before its advent.

This stratification may come about in several steps:

in brief,

– from “big man” practice
– to chiefdom spurred by warfare,
– leading to a three-class division *slave owners – commoners – slaves*.

Elman Service's equally classic *Origins of the State and Civilization* sees statal organization as the end result of a quantitative and often gradual development from

> relatively simple hierarchical-bureaucratic chiefdoms, under some unusual conditions, into much larger, more complex bureaucratic empires

The chiefdom itself was understood by Service as a hierarchical organization legitimized by social *functions* wielded by the chief for common benefit in a theocratic frame of reference, where

> economic and political functions were all overlaid or subsumed by the priestly aspects of the organization

Service thinks of functions mostly concerned with redistribution;

if we include military leadership the contrast with Fried can be seen not to be absolute.

Another oft-quoted contributor to the general debate should be singled out for relevance for the following, namely Robert Carneiro,

> in “A Theory of the Origin of the State” from 1970, and “The Chiefdom: Precursor of the State” from 1981.

Carneiro argues “what a chief gets from redistribution proper is esteem, not power”, and observes that

> As long as a chief merely returns everything he has been handed, he gains nothing in wealth or power. Only when he begins to keep a large part of it, sharing with his retainers and supporters but not beyond that, does his power begin to augment.

> But the power of a chief to appropriate and retain food does not flow automatically from his right to collect and redistribute it. Villagers freely allow a chief to equalize each family’s share of meat or fish or crops through redistribution because they benefit from it. But they will not willingly suffer the same chief to keep the lion’s share of food for himself. Before doing this, he must acquire additional power, and that power must come from some other source.

Power, then, depends on the ability of the chief to transform redistribution proper (where the chief retains only a small percentage of what passes through his hands) into *tribute* or *taxation*,

that is, into a situation where he keeps a large part for himself and for the "core of officials, warriors, henchmen, retainers, and the like who will be personally loyal to him and through whom he can issue orders and have them obeyed".

The origin of this transformation Carneiro sees in *warfare resulting from population pressure*.

Warfare is the reason that early class societies consist of *three* and not just two classes:

> The two classes that are added to a society as it develops are a lower class and an upper class, and the rise of these two classes is closely interrelated.
>
> The lower class [...] consists initially of prisoners who are turned into slaves and servants.
>
> At the same time, however, an upper class also emerges, because those who capture and keep slaves, or have slaves bestowed upon them, gain wealth, prestige, leisure and power through being able to command the labor of these slaves.

Even though considering the transition “from autonomous villages, through chiefdoms and states, to empires” as a continuous process, Carneiro finally finds it useful to distinguish the state as

> an autonomous political unit, encompassing many communities within its territory and having a centralized government with the power to draft men for war or work, levy and collect taxes, and decree and enforce laws.

Though illustrated by references to ethnographic and historical material, what Fried, Service and Carneiro offer are *general theories*.

Since their time, these theories have been tried out by specialists on a large number of single cases. This has provided many insights into the applicability of the concepts involved

> and, not least, into the historical variability of the diverse processes to which the theories make appeal.

One contribution is particularly useful for my topic, namely Henry T. Wright & Gregory A. Johnson's "Population, Exchange, and Early State Formation in Southwestern Iran" from 1975.

It focuses

> on the total organization of decision-making activities rather than on any list of criteria,

and describe the state as

> a society with specialized administrative activities. By 'administrative' we mean 'control', thus including what is commonly termed 'politics' under administration.

Further,

In states as defined for purposes of this study, decision-making activities are differentiated or specialized in two ways.

First, there is a hierarchy of control in which the highest level involves making decisions about other, lower-order decisions rather than about any particular condition or movement of material goods or people.

Any society with three or more levels of decision-making hierarchy must necessarily involve such specialization because the lowest or first-order decision-making will be directly involved in productive and transfer activities and second-order decision-making will be coordinating these and correcting their material errors.

However, third-order decision-making will be concerned with coordinating and correcting these corrections.

Second, the effectiveness of such a hierarchy of control is facilitated by the complementary specialization of information processing activities into observing, summarizing, message-carrying, data-storing, and actual decision-making.

This both enables the efficient handling of masses of information and decisions moving through a control hierarchy with three or more levels, and undercuts the independence of subordinates.

Though meant to be generally useful, this is specifically geared to what happened when statal systems emerged in southern Mesopotamia and southwestern Iran, for which reason I shall draw on it here.

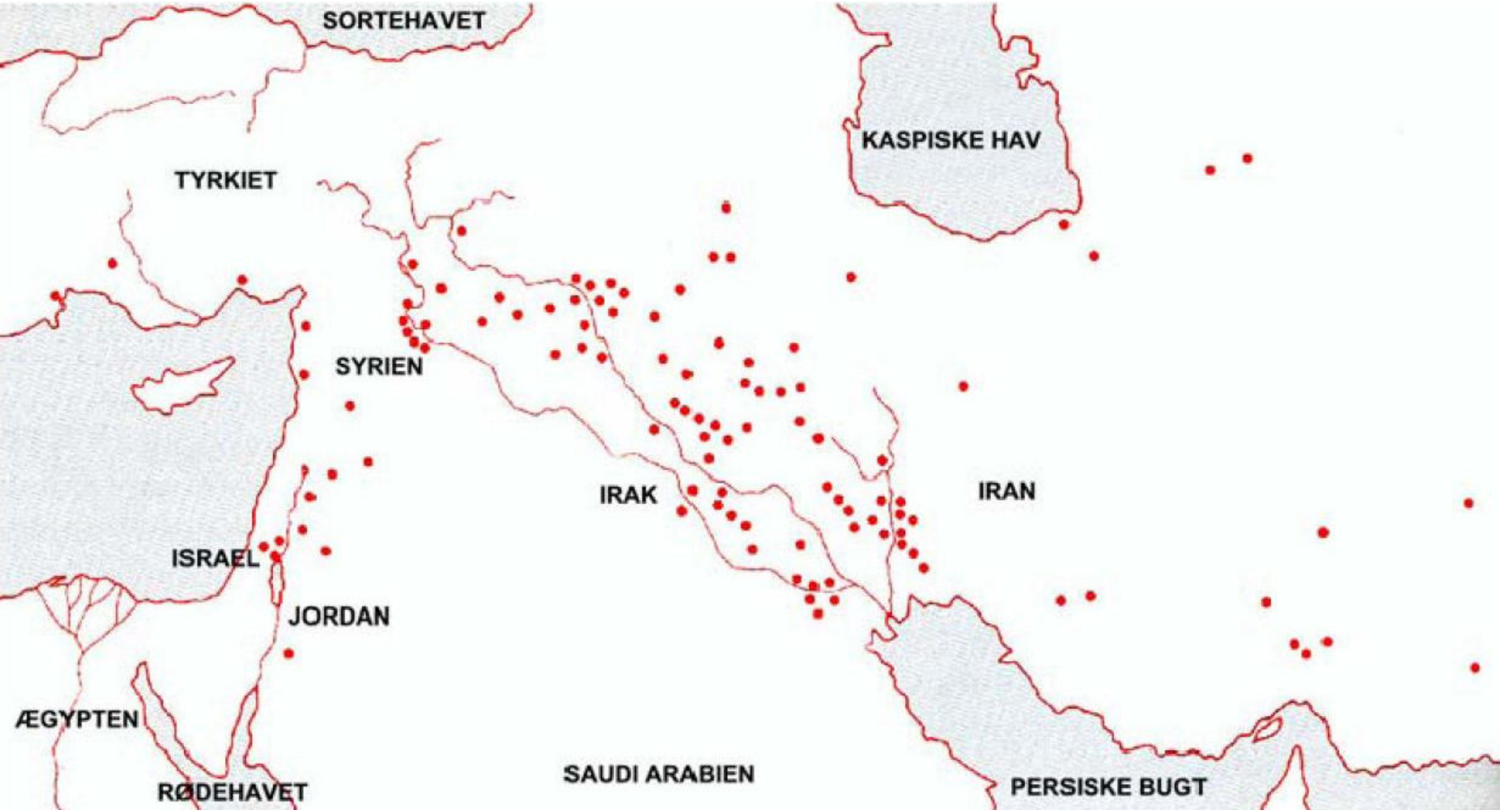
The West-Asian “token system”

Central to Wright and Johnson’s “control” is the *token system*:

an accounting system based on small and less small cones, spheres, discs, tetrahedra, rods etc. made of burnt clay – often (at first rarely) provided with markings that define sub-types.

The system turns up in Syria and Western Iran around 8000 BCE, concomitantly with the agricultural revolution,

spreading over the following millennia to a region reaching from south-eastern Anatolia and Palestine to the Indus.


SORTEHAVET
KASPISKE HAV
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RØDEHAVET
SAUDI ARABIEN
PERSISKE BUGT

It remained alive until the early third millennium BCE – in certain locations apparently until the mid-second millennium BCE.

The system and its diffusion in time and space was discovered by Denise Schmandt-Besserat.

The various tokens served to represent quantities (presumably standard containers) of grain, oil, etc., and heads of livestock – perhaps also quantities of work.

Prestige versions (made from marble, alabaster, etc.) turn up as grave-goods in high-status graves, and tokens are often found in communal storehouse areas.

This is evidence that the tokens functioned as means of accounting in a redistribution system,

and shows that management of this redistribution system carried very high social prestige – which makes it fit Elman Service's view.

Two observations:

1. Redistribution within the community is very common in pre-state societies, but redistribution built on detailed accounting is rather unique.

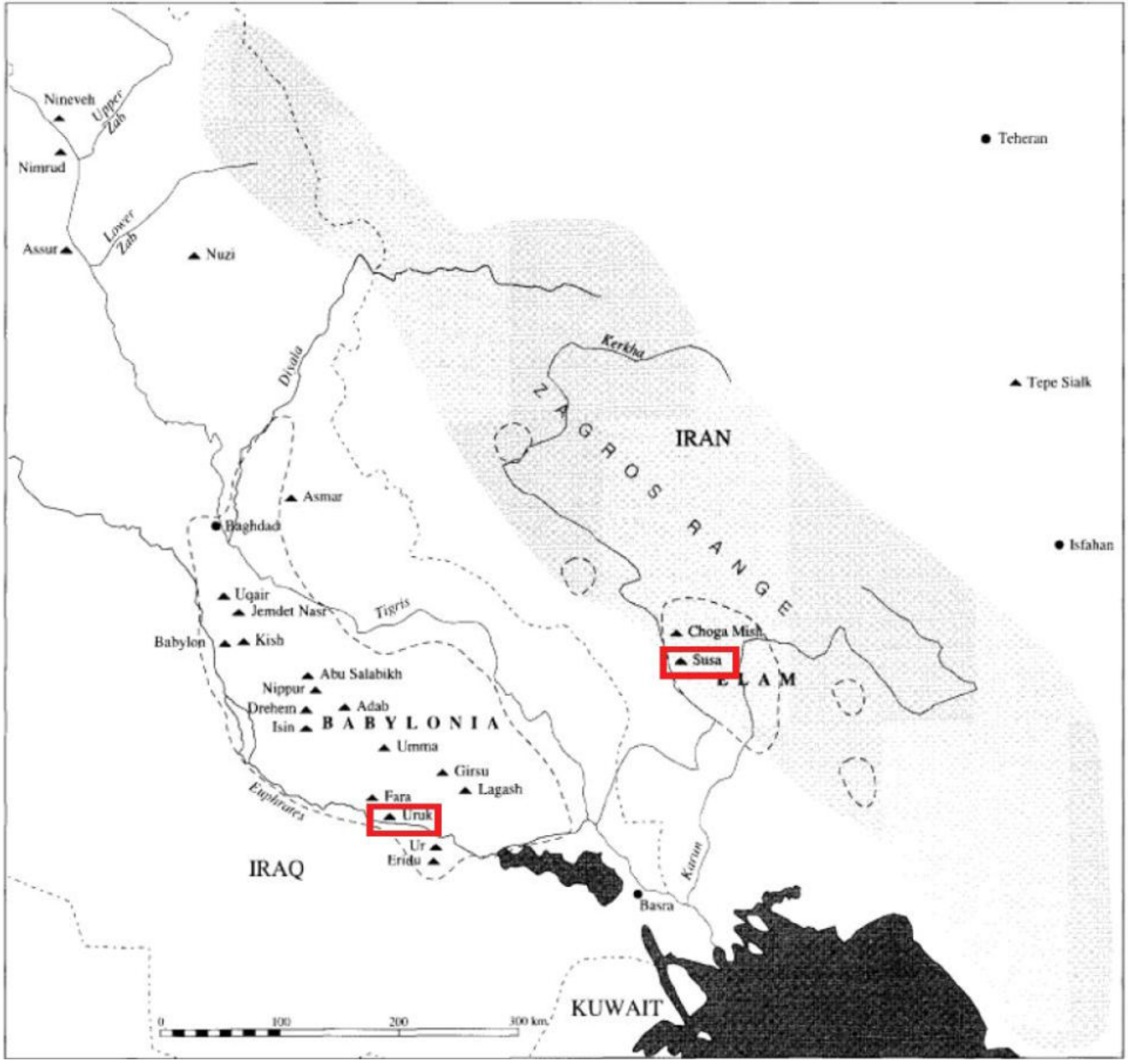
   If Inuit hunters kill a walrus and give others access to the meat, this is done from an expectation of reciprocity, and on the part of the more skilled hunters in expectation of prestige; but detailed accounting is excluded.

2. Accounting by means of tokens can doubtlessly be characterized as a *mathematical technique*. But we have no evidence for numerically standardized bundling of units (actually there is some counter-evidence).

   – It is therefore most likely that (e.g.) a small cone corresponded to a specific customary basket containing grain and a small sphere to some larger equally customary container, and that the ratio between the two was not numerically but physically (that is, not precisely) fixed.

   – i.e., the mensuration inherent in the token system was neither coordinated with the bundling levels of an oral counting system nor with any other numerical bundling principle;

   – the system is hence hardly an instance of (integrated) *mathematics*.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Upper Zab
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Lower Zab
Assur
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Diyala
Kerkha
Teheran
Tepe Sialk
ZAGROS RANGE
IRAN
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Baghdad
Isfahan
Uqair
Jemdet Nasr
Tigris
Babylon
Kish
Choga Mish
Susa
ELAM
Abu Salabikh
Nippur
Drehem
Adab
Isin
BABYLONIA
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4th-millennium developments

After 4000 BCE, the city Susa in a river valley in the Zagros area in southwestern Iran became the centre of a wider settlement system;

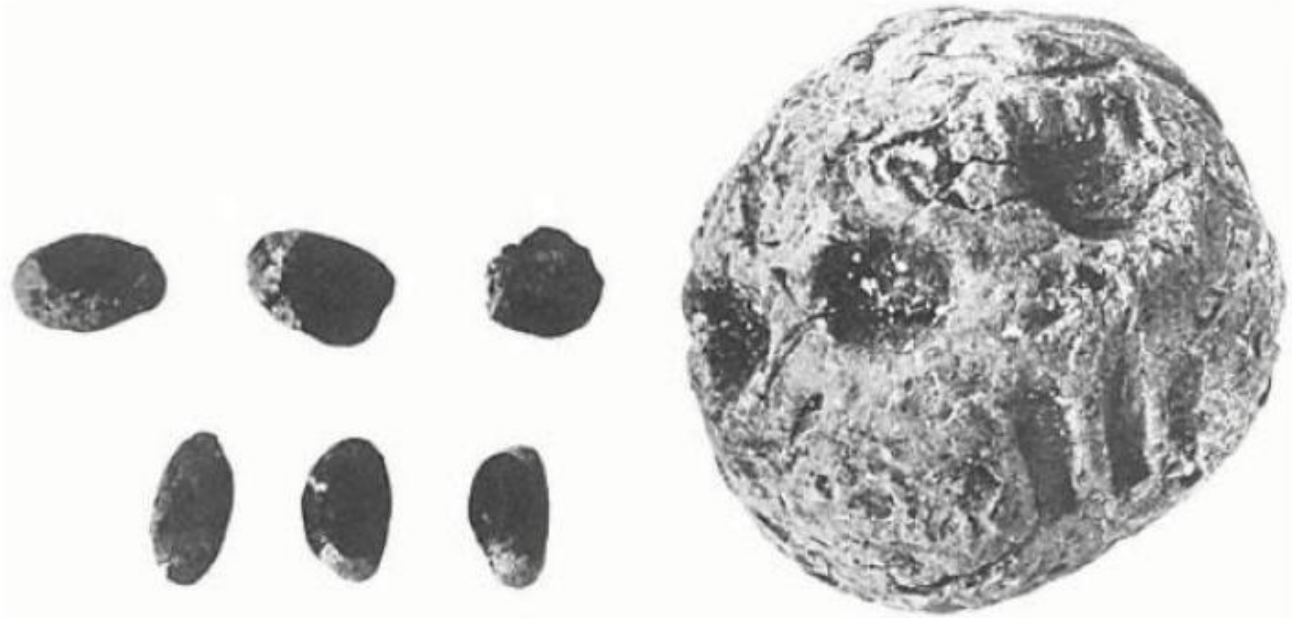
Here, the redistribution system developed into what looks most of all as payment of tribute or taxes to the central temples of Susa.

The tokens were put to new use:

> enclosed in hollow clay envelopes (“bullae”), they appear to have served as bills of lading for goods delivered from the periphery to the centre.

This goes hand in hand with the development and refinement of other bureaucratic devices and procedures – not least the use of cylinder seals as “certifiable signatures” of particular officials or offices.

As the contents of bullae could only be “read” if they were broken, impressions of the tokens to be put into them began to be made on their surfaces before they were closed and sealed.



A similar social development started slightly later in Uruk in the Mesopotamian South, but it soon went much further.

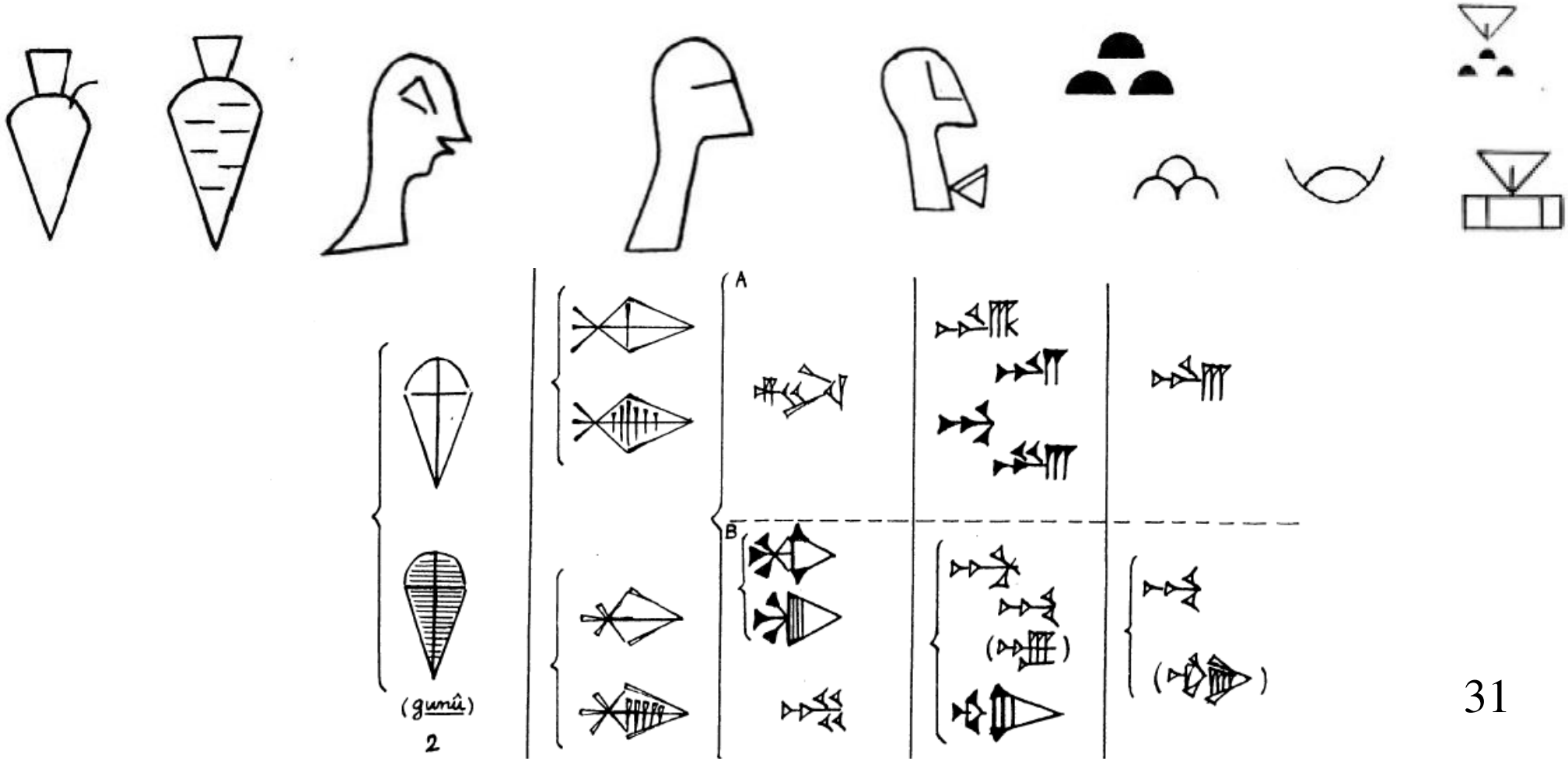
The background was a clima change and lowering of the water level in the Gulf, which allowed irrigation agriculture in the future Sumerian area, and hence a violent growth of agricultural output as well as population.

It was realized that depiction of the tokens on the surface of bullae made it possible to dispense with the contents, and that the bulla itself could then be replaced by a flattened piece of clay as carrier of the impressions/depictions.

Very soon (c. 3200 BCE), writing was also invented – *invented* indeed, in one leap or at least in a very speedy process (no "primitive" precursor steps are known).

The “proto-literate” script was ideographic. Most signs (traced by means of a pointed stylus) were directly pictographic, showing for instance a jar, a head, the mountains to the east, the sun rising between these, etc.

In the 3d millennium, the drawings were no longer traced but made by oblique impression of a prismatic stylus; this gave the script its characteristic “cuneiform” character.



A few, however, depict *tokens representing the thing* instead of the thing itself. Quite striking, and enigmatic until the discovery of the token system, is the sign for a sheep: a circle marked by a cross. Indeed, it does not depict the animal but the token standing for the animal.

In contrast to these drawings of things or tokens, metrological and numerical units were *impressed* by a different stylus, as representations of tokens.

This stylus was cylindrical, thick in one end and thin in the other. Impressed vertically it might produce a large or a small circle, oblique impression could represent a large or a small cone.

We shall encounter these signs in a moment.

The proto-literate script did not attempt to render the sentences of spoken language – it was not "glottographic".

Some 85% of the surviving texts are accounts made in fixed formats, rather to be likened to a statistical table or a ledger than to literary texts

> what was written could of course be *spoken of* or *told* in words but it could not be *read*.

The words written in these served to *procure context and meaning* for the measures and numbers – just as the headings of a statistical table.

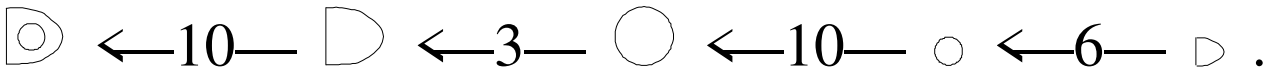
The remaining 15% are "lexical lists" which served to teach the script.

The idea and the bureaucratic use of writing (not the script itself) were soon borrowed into Susa and a number of other Iranian localities which formed a shared cultural system.

The proto-literate Uruk metrologies

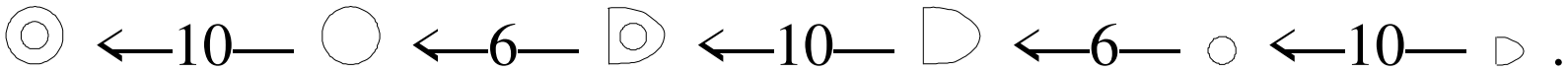
In the numerical and metrological sequences of the Uruk writing system, bundling was numerically determined.

One sequence was used for the measurement of grain, and may reasonably be considered a continuation of the traditional use of tokens. In a notation due to Jöran Friberg, the sequence as a whole looks as follows



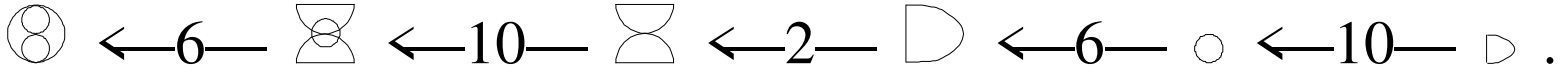
The signs will be recognized as pictures of tokens: a small cone, a small sphere, a large sphere, a large cone, a punched large cone.

Another sequence was used for counting most types of discrete items, and may be regarded as a “number sequence”.

> Whereas the grain sequence is likely to continue an old token sequence in a new medium (though now with arithmetical bundling), the number sequence can be supposed to be new.
>
> 
>

The distinct sequence of step factors suggests that it emulates an already existing oral number system, and involving multiplicative thought (thus “mathematics”).

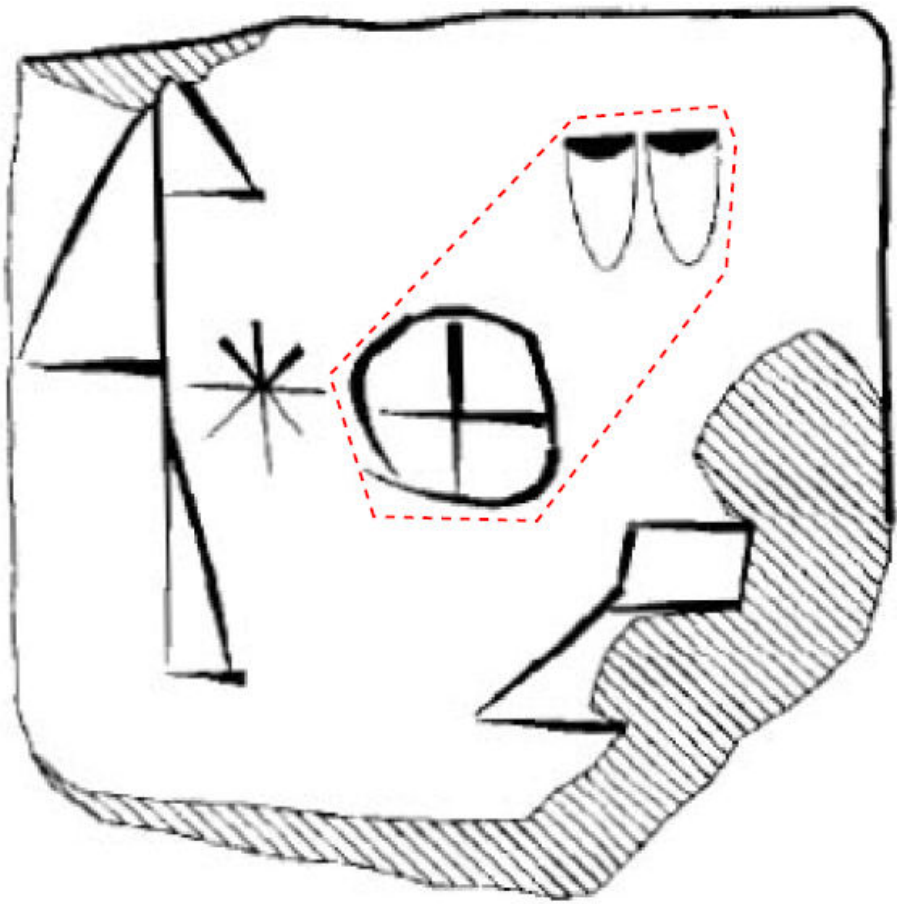
For specific counting purposes – counting of bread or grain rations etc. – a particular “bi-sexagesimal system” with the following structure was in use:



It was presumably shaped so as to fit particularly bureaucratic procedures or habits – such adaptations are amply present in the later Mesopotamian record.

There is no reason to take the existence of this system as evidence that the idea of abstract number had not yet emerged. The combination of the sign for «2» with the sign for «sheep» should suffice

– just as Engels points out that one steam engine suffices to show that the principle works.

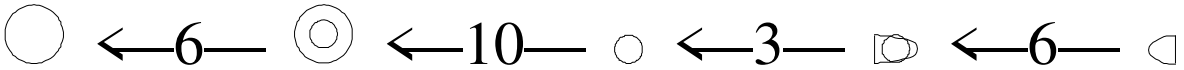


Two other metrological sequences exemplify the converse process, the adjustment of administrative procedures to mathematical structures.

– One is the area system;

– the other the administrative calendar.

The structure of the area system in itself shows little mathematical system:



Such lack of mathematical system indicates that the system is a normalization of a pre-existing system of “natural” (irrigation, seeding or similar) measures – a conclusion which is supported by linguistic arguments.

It is geared to the length metrology (based on the unit nindan or “rod” of c. 6 m).

The area metrology thus presents us

– with a deliberate coordination of several mathematical techniques;

– and with integration of the result in the administrative procedures concerned with the allotment of land in arithmetically determined proportion (which, without this new tool, could not be made, and hardly imagined).

There was a cultic, true luni-solar calendar with months of variable length and insertion of intercalary months when such turned out to be needed.

Alongside this, an administrative calendar was invented. It counted each month as if it consisted of 30 days, and each year as 12 months.

It served for the calculation of fodder to be allocated to herds and, at least in later times, of the work which overseers were to press out of their crew each month irrespective of its length.

Even in this case, only the introduction of a mathematical tool made possible the system of intense administrative control of subordinate staff.

Still other metrological sequences were in use – most of them derive from those already mentioned by means of various kinds of extra marks (similar to those that had served in the token system).

They served, e.g., to count malted instead of ordinary grain.

A *common feature of all sequences* is the way they were provided with subunits below ▷.

In all cases, the first level of sub-units was obtained by rotating either this sign, ▷, or of a shortened  , 90° clockwise, ▽ and ⌓, respectively

▽ stands for a halving (except when a day is seen as a sub-unit of an administrative month), ⌓ for a division into 5 parts.

Globally, the way sub-units are formed thus reflects an underlying general idea of “forming sub-units”.

Freely movable tokens had to represent both the *kind* of thing they stood for and the quantity involved. In writing, it became possible to separate the two, combining, e.g., the ideogram for a sheep with the number «2» – as we have seen.

The mental habit involved in this splitting of quality and quantity also underlies the principle according to which the “lexical lists” were constructed from which the script was learned.

The lists may enumerate

– various types of vessels;
– objects made from wood;
– metal objects;
– geographical names;
– and similarly.

The most elaborate of the the lists is the “profession list”, an ordered list of professions and of positions in the administrative hierarchy.

This may not seem astounding to us – we categorize the world more or less in the same way. But not everybody does.

This was shown by the Soviet psychologist Aleksandr R. Luria in the 1930s.

His work with illiterate peasants and young kholkoz activists with some schooling to introduce a distinction between “situational thinking” and “categorical classification”.

The contrast is illustrated by this dialogue:

Luria, explaining a psychological test: “Look, here you have three adults and one child. Now clearly the child doesn’t belong in this group”.

Rakmat, an illiterate peasant from Central Asia: “Oh, but the boy must stay with the others! All three of them are working, you see, and if they have to keep running out to fetch things, they’ll never get the job done, but the boy can do the running for them [...]”.

Luria found situational thinking to be “*the* controlling factor among uneducated, illiterate subjects” in his research on cognition in Soviet Central Asia;

“Young kolkhoz activists with only a year or two of schooling”, on the other hand, employed the principle of categorical classification “as their chief method of grouping objects”.

Categorical classification – what we find in the lexical lists – is abstract and analytical in the sense that it looks at things in isolation from the wider situation in which they occur or are used.

Situational thought, instead, understands the world synthetically, through total situations; it is "economical" for a person whose life is made up by fixed and recurrent situations.

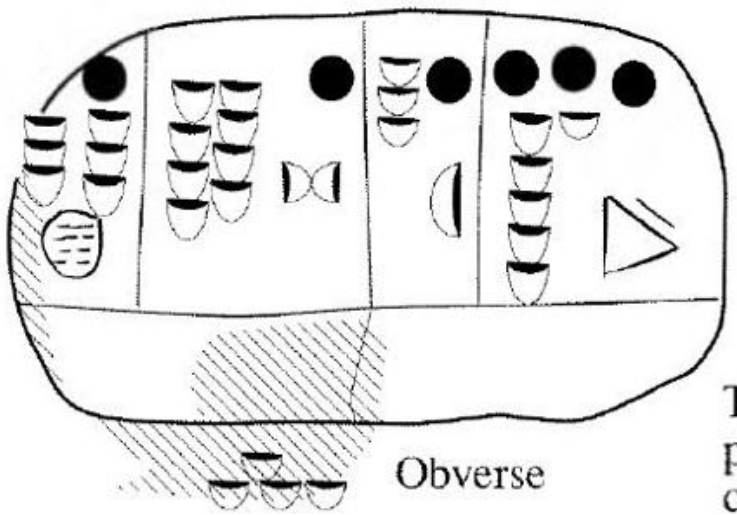
Categorical classification, on its part, is the adequate way to orient oneself in a life world which is not organized in this stable way

– it was adequate for temple managers who had to think of a plough both as an object to be constructed in the workshop, as an agricultural tool, and perhaps as an object of taxation.

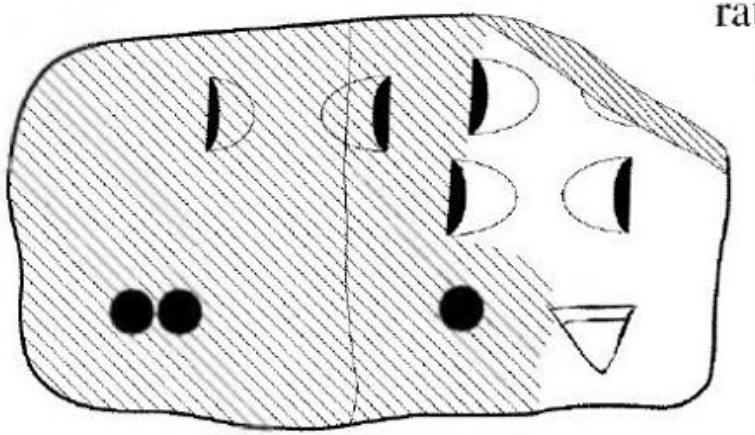
As a whole, the system of lexical lists is a kind of “Cartesian product”: along one dimension we have the categories, each in its own list. Along the other, the elements of the category.

In the “profession list” the Cartesian product is not only an external condition but also involved in the structure of the list itself, which confronts field of activity with the hierarchy of positions (leader – foremen – workers).

Even the orderly formats of bureaucratic accounting reflects the same mental habit.



Obverse



Reverse

Tablet with four entries on the obverse, a notation on the edge and possibly a summation (?) on the reverse (damaged): The obverse contains entries concerning various grain products. The entries on the reverse are collectively designated (NINDA), i.e., "grain rations." Possible reconstruction of the total:

$5 \times (35 + 13 + 18 + 16) = 410$ (i.e., $3 \times 120 + 5 \times 10$)

The lists were a means for teaching, and thus a vehicle not only for literacy but also for the »modern«, abstracting mode of thought – precisely the mode of thought preferring mathematical coherence to situationally adequate seed measures, etc.

The latter part of their message will have supported, and have been supported by, the development of the main administrative tool: the clay tablet with its ordered formats

Uruk: A “mathematical state”

If mathematics proper is understood as

> the coordination of *pre-existent but previously independent* mathematical practices [...] through a minimum of at least intuitive understanding of formal relations

then there is no doubt that mathematics arose, if not before, then at least in late preliterate or in proto-literate Uruk.

The primary background was the needs of the administration of the new social system that asked for the creation or further unfolding of mathematics.

Conversely, the bureaucratic use of the mathematical tool was no instance of pure “technical rationality”, the creation and implementation of means for an already established end which itself is not touched.

If we compare the Uruk state formations with other early states, the end itself (the state) can be seen to have been shaped by the means (mathematics), just as the successful appeal to military means may lead to the transformation of the state that appealed to it.

Substantiation:

Redistributive systems are widespread in pre-state societies; they correspond to the need for mutual support, and may thus be said to correspond to a notion of social justice.

However, this notion of justice cannot easily be carried over to the proto-statal situation.

We remember Carneiro's words,

> what a chief gets from redistribution proper is esteem, not power;

and further,

> As long as a chief merely returns everything he has been handed, he gains nothing in wealth or power. Only when he begins to keep a large part of it, sharing with his retainers and supporters but not beyond that, does his power begin to augment.
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But they will not willingly suffer the same chief to keep the lion's share of food for himself. Before doing this, he must acquire additional power, and that power must come from some other source.

Power only results when redistribution proper (where the chief retains only a small percentage of what passes through his hands) is transformed into *tribute* or *taxation*, where he keeps a large part for himself and for the

> core of officials, warriors, henchmen, retainers, and the like who will be personally loyal to him and through whom he can issue orders and have them obeyed.

Neither the commoners nor the chief and his circle have any immediate reason the conceptualize the new situation in terms of social justice.

In the Susa-Uruk area, matters were probably perceived differently, even though realities may have been similar.

Bullae and accounting tablets show that taxation and allocation of resources – fields apportioned to high-ranking temple officials, rations of grain distributed to workers – were made according to mathematically determined rules.

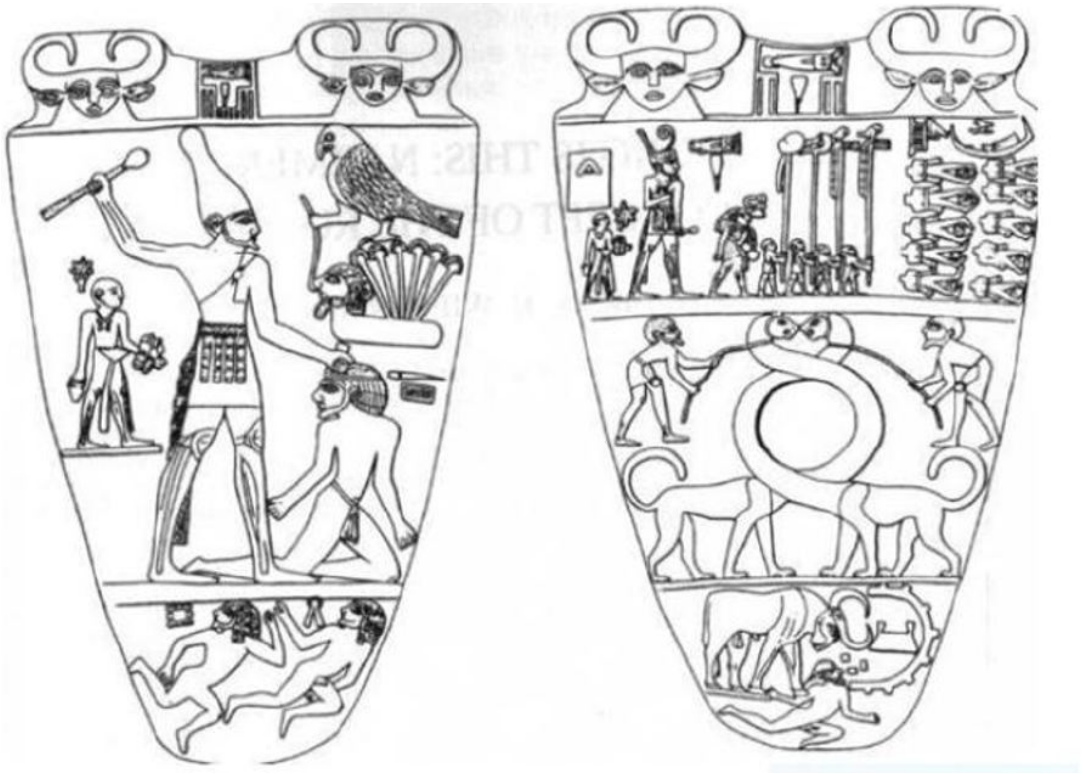
Statal power was hence structured around “just measure” and thus, apparently, legitimized by a transformed concept of social “justice”.

Since accounts and lexical lists constitute our only written sources, we have no direct evidence for how the situation was conceptualized at the time;

however, literary evidence from a time when lexical lists from the proto-literate period were still in use indicates that at least the higher literate stratum thought of statal power in such terms.

A striking contrast is provided by early Pharaonic Egypt.

The Pharaonic state was apparently legitimized by conquest, and (at least in the view of the literate) by a religious guarantee of cosmic order. The earliest surviving *numbers* boast of booty.



Already during the First Dynasty, it is true, the yearly level of the Nile was recorded, in all probability in order to allow calculation of the taxation level of the year to come,

and a biennial “counting of the riches of the land” was introduced.

But a biennial counting certainly does not allow any specific determination of dues and rights, nor is there any evidence that the measured Nile height served such purposes.

Social “justice” has no place in the picture of early Pharaonic Egypt.

To what little I know, Shang Dynasty China was more similar to Egypt than to Uruk.

“Real justice”

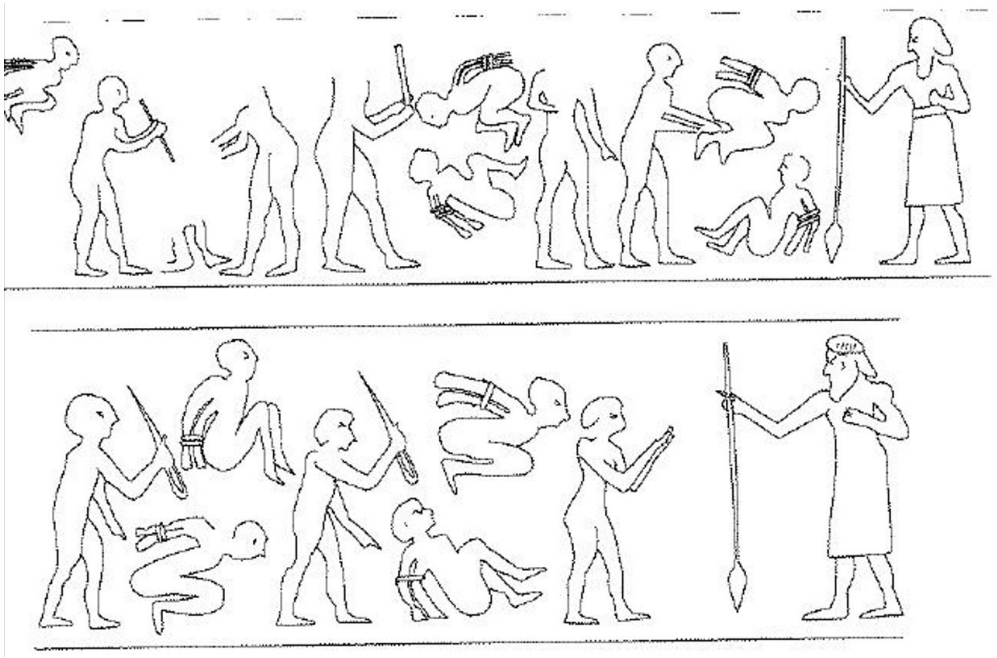
- Soviet “Real socialism” did not coincide too well with what had been proclaimed in programmes;
- the real feudalism of the Middle Ages was conspicuously different both from Charlemagne’s blueprint and from the ecclesiastical theory of the respective roles of the praying, the warring and the labouring order.

Likewise, mathematical social “justice” (however much unequal) was certainly not the whole truth about the Uruk state.

But it remains an essential part of the truth, and it conditioned Mesopotamian statal structures at least until the mid-second millennium BCE.

On the other hand, only *part* of the truth, belonging rather on the level of hegemonic ideology than on that of social realities.

This follows from the preferred motif of the seals of high officials (found on no less than half of all known early Uruk seals: A high official or priest looking on while overseers beat up pinioned prisoners.



(should we say, the beginning of “Western civilization”?)

It is probable that the violent increase in population did not result from local breeding alone but also from enslavement of significant populations from the mountain areas to the east.

- The pictograms for male and female slaves are indeed composed of an indication of sex (of a person) with a picture of the mountains

- It is supported by linguistics: many features of Sumerian look like those of languages that over some centuries have developed from pidgins and creoles.

  > In the long run, masters taken care of as children by lower-class nurses and servants will have adopted the creole (probably without perceiving the shift as a change of language) – the final outcome (after centuries) being Sumerian.

10
1 minutes' break

The Early Dynastic and Sargonic periods, 2900–2200

The proto-literate period may have lasted from c. 3200 BCE to c. 2900 BCE.

It was followed in the Sumerian area (now doubtlessly Sumerian) by the "Early Dynastic Phase", c. 2900–2350 BCE (habitually abbreviated ED, subdivided into ED I, ED II and ED III).

A social system with one major centre (Uruk) changed (collapsed?) into one consisting of competing city states;

and a state centred around a staff of high temple officials developed into states rules by a king (a war leader).

Written sources are rare before 2600 (ED I–II) but proliferate from then one. The continued use of the old lexical lists demonstrates continuity not only of the writing system but also of the school tradition.

However, from 2600 writing was in wider use, serving also, e.g., for the stipulation of private contracts;

concomitantly, and in consequence, the circle of the literate became broader – a transition from “very restricted” to “restricted literacy” takes place.

The group of scribes (dub.sar) turns up for the first time as a distinct *profession* in the city-state Shuruppak (c. 2550).

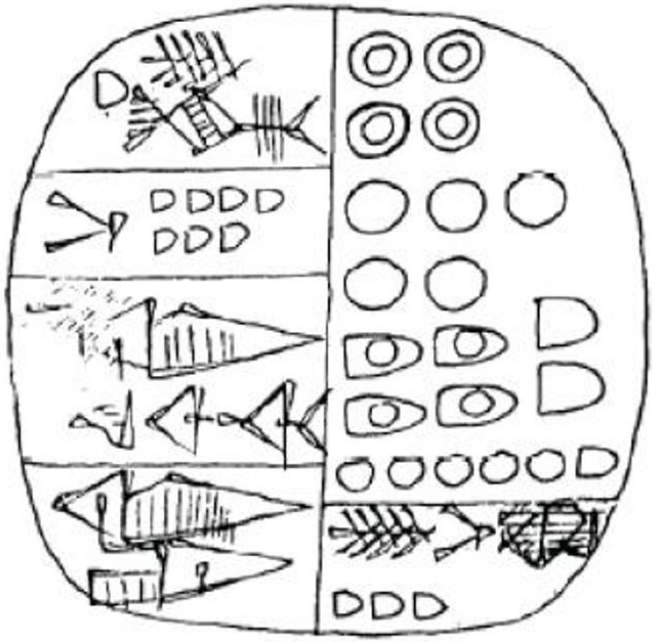
At the same time, and in all probability as a consequence of this, the script was put to new uses.

– We find the first literary texts – a proverb collection and a hymn

– and the first instances of “supra-utilitarian” mathematical school problems (problems that are not directly connected to practice even though they are formulated as if they were).

One of them runs like this:

Grain, 1 granary.
7 SÌLA each man receives.
Its men?
164,571.
3 SÌLA left on the counting board.



A granary is not only a physical construction but also the largest metrological unit – the largest "round number" in grain measurement, consisting of 40×60 "tuns", each of which contains 8×60 "litres" (SÌLA).

7 never occurs as a divisor in actual administration; it is more difficult than what is needed in bureaucratic practice because it does not divide the metrological step factors.

164,571 men exceeds the total population of the city.

This is, if not “pure”, then inapplicable mathematics.

In contrast, all mathematical texts from the proto-literate period that can be identified as school texts are “model documents”,

> distinguishable from real administrative texts only by the absence of an office seal and by the occurrence of numbers that are suspiciously round, nice or large.

Literary texts as well as supra-utilitarian mathematics were probably meant to probe and make manifest the reach of the two professional tools – writing and computation – and thus as expressions of professional pride.

Rising city walls show clearly that warfare was an endemic condition of the ED-period, and that the king was a military leader.

Shuruppak itself was completely devastated in a military attack, following upon a general mobilization.

The many killed servants that followed their master to the underworld in the Royal Cemetery of Ur (c. 2600) demonstrate that the king had left behind any idea that he was the servant of society – he was its overlord, and society was a means for his greatness.

None the less, only the very end of the ED period gives us written evidence, if not of the ritual slaughter of servants then at least of military activities;

> until then, even royal inscriptions show the king solely as the benefactor of temples and provider of agricultural prosperity.

Literacy, so it appears, only reflects the functional and pseudo-just characteristics of the state.

Those features of the state which had been irrelevant for the invention of writing and bookkeeping remained outside the perspective of writing.

In this respect, ED Sumer was a *dual society*, one of whose faces was still “mathematical”.

This concept calls for some sketchy and ultrashort comparative remarks.

Latin Europe of the Central Middle Ages (*ca* 750 to *ca* 1050 CE) tended toward this kind of duality.

The cathedral schools aimed at procuring administrative staff, and almost all writing was bound up with administration, administrative record-keeping and religious or ecclesiastical matters.

From the moment we know about writing in Egypt (the time of Narmer, the beginning of the first dynasty) and China (the Shang), writing gives testimony of state violence. “Duality” is an irrelevant concept.

That ED III was a dual society is thus a *distinctive characteristic,* resulting from the particular intertwinement between administration, writing and state formation in southern Mesopotamia.

> Just as the semi-dual character of central medieval society came from the specific survival of ancient literate culture carried solely by the Church at the collapse of Western Rome.

From c. 2350 to c. 2200 BCE, the Sumerian area, and soon the whole of Mesopotamia was united into a single territorial state;

after an initial short-lived centralization around a Sumerian city-king, the centre was the Akkadian "Sargonic" state

> (Akkadian is a Semitic language, Sargon the founder of the dynasty; the school language remained Sumerian).

"Literature", originally a free creation of the scribe school and a means for scribes to probe and demonstrate their professional identity, was soon taken over by the Sargonic rulers as propaganda.

Mathematical administration expanded, and the use of supra-utilitarian problems in mathematics teaching was continued;

> there is no reason to presume that they fulfilled, or could fulfil, any role outside the school.

Accelerating during the Sargonic period, metrologies were adjusted with concern for mathematical regularity as well as administrative convenience.

The former concern (mathematical regularity) is especially visible in the weight system, a fresh development of the ED phase

Here, the step factor 60 was given a prominent position.

But even older metrologies were extended upwards and downwards with the factor 60.

The concern for administrative convenience, at times but not always in conflict with the former, asked for the adaptation to administrative procedures or technical practice

– One example is the introduction of a Sargonic “royal” gur (“tun” – the largest capacity unit)

– another the creation of particular brick metrologies geared to the various standard bricks.

All in all, the relation between the state and its mathematics developed during the later ED and the Sargonic period along lines known from other societies provided with an accounting or otherwise mathematically organized administration:

– mathematics was taught in a way which was needed by future staff;

– but it was also allowed a certain autonomy in the school.
Certainly not taught by "mathematicians" – but even when teachers are supposed to teach for practice, teaching will normally be affected by the fact that the practice which teachers are *really* familiar with is the practice of teaching.

The Janus-faced innovations in metrology correspond to this tension in the situation of mathematics:

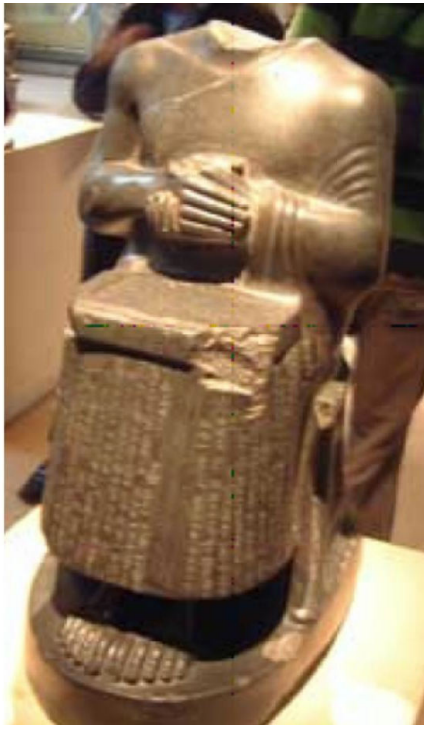
– “sexagesimalization” driven by a the search for mathematical coherence;
– the other innovations by the links to extra-mathematical practice, in particular in the administrative procedures of the state.

Gudea and the Neo-Sumerian state

Around 2200, the Akkadian territorial state or "empire" lost most – in the end all – of its territory, and smaller states reemerged.

Gudea's Lagash (2141–2122 BCE) has left sources that might be considered relevant for our topic:

inscriptions telling in meticulous accounting what king Gudea has given to the temple, and how he laid out the geometric plan for sacred buildings.



From 2112 BCE onward, however, the Third Dynasty of Ur established a new “Neo-Sumerian” territorial state or empire, mostly referred to as “Ur III”.

The early decades of this dynasty present us with nothing spectacular. In 2074 BCE, however, king Šulgi undertook a military reform, which was immediately followed by an administrative reform.

From this point onward, hundreds of thousands of accounting tablets inform us about the details of the administration.

When the city burns, papyri and paper in
its libraries burn with it.

Libraries containing clay tablets fare better:
The wooden shelves burn, and the tablets
may break when they fall – but they
survive.

At least in the core area, the larger part if not the overwhelming majority of the working population in both agriculture and handicraft production seems to have been submitted to conditions close to those of slavery.

Workers were organized in crews under scribal overseers who were responsible for the work performed, reckoned in units corresponding to $^{1}/_{60}$ of a working day (i.e., 12 minutes), according to pre-established norms.

The accounts of the overseers are extremely meticulous, taking illness, death and absence as well as workers lent to or borrowed from other overseers into account.

The old administrative calendar was still in use – Ur III is the epoch in which sources show that the overseer scribes were to press out of their crew 30 days' work each month irrespective of its actual length.

As shown by the late Robert Englund, the yearly deficits of an overseer scribe were accumulated, and at his death the family was held responsible for it (if needed by being drawn into the enslaved crews).

For use in this immensely expanded accounting, two decisive mathematical innovations were introduced.

One is the accounting system itself, with built-in automatic controls (much like double-entry bookkeeping).

This was taken over in the subsequent “Old Babylonian” period, during which it was also used for private large-scale accounting – and then forgotten.

The other was the sexagesimal place-value notation.

A floating-point system, serving equally well for integers and for fractions.

– Not useful in accounts or other economic documents, where the order of magnitude is decisive (60 šekel or $^{1}/_{60}$ šekel of silver!).

– Used instead for intermediate calculations, of which relatively few traces remain;

– in mathematical school texts, where orders of magnitude could be presupposed, could be remembered, or were immaterial;

– and in the late astronomical tables, where the tabular format helped to determine orders of magnitude.

Neither school texts nor astronomical tables can have been the original purpose for which the system was introduced – the latter already for chronological reasons.

Nor did it ease additive and subtractive computations (which anyhow appear to have been performed on some abacus-like device).

What it did facilitate was multiplication and division

– but only if multiplication tables and tables of reciprocal numbers were available or learned by heart,
– along with tables permitting the translation of metrological units into sexagesimal multiples of a standard unit.

The production and teaching of such tables, on the other hand, had no point before the place-value system was adopted.

The striking conclusion:

The important step was not the *invention* of the new notation

– as shown by Marvin Powell, it had been in the air since centuries
– it may even have been invented well before Ur III without leaving any traces in tablets that happen to have survived and to have been read by Assyriologists.

What was decisive will have been a *political decision to implement it* – a decision which could only be effectual in a centralized system like Ur III.

We have no direct evidence for the time and place of such a decision;

but we may safely assume that the planning was made in a scribe school environment that was closely connected to the royal administration.

That king Šulgi himself (or at least his ghost writers) saw the school as an essential tool for his project is obvious from one of the so-called Šulgi hymns:

The king boasts he was taught from an early age in the "tablet-house",

– learning the art of writing together with addition, subtraction, counting and accounting under the protection of the scribal goddess Nisaba;
– his praise is song in the same tablet house.

Considering the marvellous feats of which Šulgi brags elsewhere in this and other hymns we may wonder at the elementary level of the mathematical curriculum

– even multiplication goes unmentioned.

Actually, this fits what can be derived from the absence of all mathematical school texts apart from model documents, in particular when viewed in the light of evidence offered by the terminology of the Old Babylonian period.

Problems, well represented in the (meagre) corpus of mathematical texts surviving from Shuruppak and the Sargonic period, were banished from the Ur III school:

> even the modicum of independent thought needed if students have to find and not just follow a prescribed way seems to have been considered a threat to their docility.

The "Šulgi hymns" and the prologue of the law-code he produced are informative about the official ideology of the state.

– Šulgi is not only a potent military leader and pitiless avenger of wrongs (which, conveniently, permits him to provide slaves)
– but also a "good shepherd" and exceedingly just (dual society, passed away in late ED III, had not been resurrected).

However, only one feature of his "social justice" goes beyond verbatim repetition of the trite commonplaces of the preceding centuries (protection of orphans from wealthy and widows from mighty men), and only one thus rings true: metrological reform.

(Social justice should be distinguished from "judicial justice", punishing enforcement of the laws which follow after the prologue.)

Ur III thus enhances features which already appeared to characterize proto-literate Uruk:

The management of the state was meticulously planned and controlled, which had several effects:

– In mathematics, important innovations were introduced – one of them (the place value system) still important for us.

– Free supra-utilitarian developments, on the other hand, appear to have been blocked.

– Socially and ideologically, the fact that the extremely oppressive policies of the system were metered out according to mathematical rules permitted that these could be seen by those in power – and probably even by the overseer scribes – as exponents of *justice*.

The undernourished workers, however,

– fell ill or ran away the best they could – even this can be read from the accounting texts;

– after all, they had not been brought up in the scribal school and may have had other opinions about social justice if at all caring about such questions.

An Old Babylonian epic poem seems to reflect large-scale strikes during Ur III.

This is likely to be one of the reasons that the Ur III state did not outlast the third millennium.

The Old Babylonian period and the culmination of Mesopotamian mathematics

Around 2025, the periphery rebelled, and the Ur III state lost its character of an empire.

A few decades later, even the centre dissolved into small states. Gradually, some of these absorbed the others, and in the 18th century BCE Hammurabi of Babylon managed to subdue the whole Mesopotamian south and centre.

From then on, this region can be spoken of as “Babylonia”; the centuries from 2000 BCE to 1600 BCE are known as the “Old Babylonian period”.

This period produced the most sophisticated mathematics we find in ancient Mesopotamia.

NB: Not the *most sophisticated mathematical astronomy*. Mathematical astronomy only started around 700 BCE.

This culmination arrived when the all-encompassing mathematical management of Ur III had disappeared.

The OB period is characterized by individualism, both in the economic structure and on the level of ideology or culture:

– Land, even when owned by the Crown, was often rented under contract.
– Private correspondence turns up. The letters were often written by free lance “street corner” scribes (a category we do not know from Ur III).
– The Ur III accounting system was now used in private business, handled by privately employed scribes.
– The seal, so far a symbol of office, now belonged to the individual.

We may speak of the rise of an ideology of *personal identity*.

This ideology also affected scribal culture, in a way which is reflected in the texts used in school to inculcate understanding of what should characterize a *real* scribe ("examination texts").

The Sumerian language was dead by now, and Babylonian could be written adequately with a phonetic syllabary of 70 signs or less (some laymen did);

– a *true* scribe, however, would also use a large number of word signs, borrowed from the Sumerian script but now meant to be pronounced in Babylonian.

– This, however, was not a sufficient demonstration that the scribe was somebody special.

– He should also be able to read, write *and speak* Sumerian
(a feat only other scribes would be able to appreciate).

– He should know everything about bilingual texts, he should be familiar with all the significations of the cuneiform signs
(each single sign would have one or several phonetic and one or several logographic meanings – to which comes further occult meanings).

– He should know about music,

– and about mathematics.

The whole complex was called “humanism” (true! – namely nam-lú-ulu, Sumerian for “the condition of being human”).

The texts do not specify *which* kind of mathematics would count as "humanist".

Training tablets which carry a Sumerian proverb on the obverse often have quite simple calculations on the reverse. Simple mathematics was thus taught at the advanced level, and most scribes presumably never went further.

On the other hand, however, very sophisticated supra-utilitarian mathematics was also produced, and it is a fair guess that this (as useless as spoken Sumerian) was the really "humanist" level of mathematics.

What we find together with Sumerian proverbs are elementary numerical multiplications, area determinations and such things.

Before that, future scribes copied metrological tables and tables of reciprocals and multiplication – probably so often that they learned them by heart.

All of this was useful training for future professional practice, and hence not supra-utilitarian.

At the sophisticated, supra-utilitarian level we still find numerical problems:

– for instance, an intricate technique for finding reciprocals of numbers not listed in the standard table nor easily derived from it by successive halving and doubling.

The favourite genre, however, was what has been interpreted as “algebra” of the second (at times the third) degree.

Nominally, these “algebraic” problems deal with areas of rectangles or volumes of excavations and their sides, at times combined for instance with the wage to be paid for the excavation (the “algebraic” *technique* was also geometric, not numerical).

The substance of the problems, however, is entirely artificial, and the "algebraic" technique that is taught completely useless for professional practice.

This "algebra" is thus truly supra-utilitarian.

The inspiration for this geometric proto-algebra had probably come from a riddle tradition carried by "lay", that is, non-scribal, Akkadian-speaking surveyors.

These riddles (as they can be reconstructed from consideration also of their appearance in much later surveying texts) were of this kind:

– "I have added together the side of a square and its area, and the outcome was 110".
– "I have added together the four sides of a square and its area, and the outcome was 140".
– "I have added together the length and the width of a rectangle, and the outcome was 14, while its area is 48".
– "the diagonal of a rectangle is 10, and its area is 48".
– "I have added together the diameter, the perimeter and the area of a circle, and the outcome was 115".

Others probably concerned *differences* between square area and one or all four sides, the sum of or difference between areas and sides of two squares.

The total number of the riddles will not have exceeded ten to fifteen.

As stated, mathematical *problems*, and *a fortiori* supra-utilitarian problems, appear to have been totally absent from the Ur III school.

As the Old Babylonian scribe school developed, its "humanist" pretensions appear to have induced it to adopt these riddles.

In the context of the school, however, a handful of standard riddles could not do; these would easily be learned heart.

Instead, the riddles became the starting point for a genuine mathematical discipline, with rich variation and exploration of the possibilities offered by the technique

- for instance, letting the sides of a rectangle represent a number or a price, or even a square area or the volume of a cube (in a problem of the 8th degree).

Beyond their “humanism”, scribes were (supposed to be) proud, if not of being leading officials of the state (few of them of course were), then of belonging to a group from which leading officials came.

This state was still supposed to represent social justice, and serving it could hence be a reason for pride.

However, the statal social justice of which Hammurabi proclaims himself the supreme protector in the introduction to his famous “law-code” was not mathematical but a continuation of commonplaces going back to the outgoing Early Dynastic epoch:

Hammurabi is still the protector of orphans and widows.

Beyond that his justice is judicial.

One of his successors also issued a decree “re-establishing justice to the country”, prescribing a debt cancellation,

– apparently a once-only measure meant to palliate the threat to general economical stability resulting from a debt crisis and crushing interest rates;
– in any case a cancellation of the very idea of that “mathematical justice” where everyone receives and contributes his exactly calculated due (indeed the “justice” which had led to the crisis).

Accounting, was still around, but even when done for the state its role was that of a subservient tool.

The relation between the state and mathematics had become accidental, not constitutive for either part.

Disappearance of a pattern

The final dissolution of the pattern state-social justice-accounting mathematics arrived with the collapse of the Old Babylonian state.

After a Hittite conquest of Babylon (1600 BCE) and ensuing social chaos, power was taken by the Kassite tribes, already present in Babylonia as labourers and mercenary soldiers.

– The ratio between town and countryside dwellers fell to 5th-millennium levels,

– and the role of scribal administration and culture – always the carriers of ideas of the just state – was not only strongly reduced but also lost its ideological hegemony.

As the Assyrian city-state expanded into a territorial state and finally an empire (12th to 7th c. BCE), writing once again became copious.

Scribes were again found in central positions at court or somehow working for the court

– but now as producers of an ideology emphasizing the king and the empire as creators and upholders of order,

– and as omen priests and astrologers protecting the king.

The letters from these scholar-scribes, apart from giving technical advice, mostly wish the king good health and vigour.

One exorcist needed to flatter Aššurbanipal, the last major Assyrian ruler (668–627 BCE).

> Aššurbanipal, in contrast to his predecessors, was pleased to take up themes from earlier epochs
>
> > (perhaps because he had at first been trained to become a high priest and not a ruler)

The exorcist therefore praises him for having brought prosperity to the land (Aššurbanipal does as much himself, as Hammurabi had done 1100 years before);

– and for distributing *particular* favours;

Even when trying to appear in the light of age-old traditions, the Assyrian king could only taint Iron-Age despotism with commonplaces of (mainly judicial) justice in Old Babylonian style.

Aššurbanipal as High priest

The scribes working for or corresponding with the court were certainly proud of their professional status.

However, even those of them who worked on mathematical astronomy identify themselves as “writers” of omen series, exorcists etc.;

their mathematical feats appear to have been peripheral to their professional self-esteem.

Ordinary daily administration was probably taken care of in Aramaic alphabetic writing, and not in cuneiform on clay tablets

(for this reason the evidence has disappeared together with almost all traces of the clerks who took care of it).

In any case, this very absence of the ideologically central language and writing style from accounting shows it to have been subordinate.

To sum up: During the late 4th and the 3d millennium, “writing” was in power;

but “writing” in this respect was first of all accounting and management of resources, somehow connected to the pre-historic redistributive structures.

After 2550 we then find the first evidence of literary writing and supra-utilitarian mathematics as evidence of professional self-esteem of scribes, and soon afterwards the use of literature as state propaganda.

During the Old Babylonian period, the role of professional self-esteem becomes much more conspicuous in scribal culture; concomitantly, the legitimization of the state, though still referring to "justice", is decoupled from accounting.

After the Kassite interlude (the "Babylonian Middle Ages"), "justice" however meant does not characterize the role of the state;

> activities of importance for the professional self-esteem of cuneiform scribes were predominantly literary, divinatory and theurgical.

The "Wittfogel thesis"

In 1957, Karl Wittvogel's notorious book on *Oriental Despotism* presented a simple thesis connecting the first development of mathematics and astronomy with the rise of the early "Oriental" state

— namely that the state was "hydraulic", i.e., developed in order to plan large-scale irrigation, and that mathematics and mathematical astronomy were created for that purpose.

Therefore, *all* "Oriental states" are despotic and enemies of "Western democracies".

First of all, of course, the USSR and the PRC.

I quote:

(A) The need for reallocating the periodically flooded fields and determining the dimension and bulk of hydraulic and other structures provide continual stimulation for developments in geometry and arithmetic. [...] Obviously the pioneers and masters of hydraulic society were singularly well equipped to lay the foundations for two mayor and interrelated sciences: astronomy and mathematics.

As a rule, the operations of time keeping and scientific measuring and counting were performed by official dignitaries or by priestly (or secular) specialists attached to the hydraulic regime. Wrapped in a cloak of magic and astrology and hedged with profound secrecy, these mathematical and astronomical operations became the means both for improving hydraulic production and bulwarking the superior power of hydraulic leaders.

The thesis is still widely held, though often in less outspoken and rigid form.

For this reason, it may be useful to look at the accuracy of this piece of Cold-War vulgar Marxism, written when Wittfogel was working as a secret informer for McCarthy and the FBI.

If we concentrate on Mesopotamia, Wittfogel is wrong on all factual accounts (Egypt would come out no better, nor probably China).

(1) Irrigation systems only became a bureaucratic concern (and then only in certain periods) many centuries after the rise of statal bureaucracy.

No doubt the irrigation economy provided the surplus needed to feed the bureaucracy; but it was taken care of locally, and often by kin-based communities (as it often is even in today's Iraq).

(2) Old Babylonian mathematical texts deal with construction of irrigation works,

> but only with the need for manpower, the wages to be paid, and the volume of earth involved. The dimensions of the constructions were not determined mathematically.

(3) Neither the sacred nor the secular calender were ever involved in irrigation planning in Mesopotamia.

(4) Mathematical astronomy was only created almost 3000 years after the rise of the state, and was concerned with the moon and the planets, i.e., irrelevant for irrigation planning.

(5) Even astrology is a late invention. This too had nothing to do with irrigation but served to give warnings about threats to the health of the ruler and to the safety of the kingdom.

(6) bureaucratic computation and occult endeavours of any sort were never connected through a common group of practitioners.

When occult endeavours became a main interest for scholar-scribes in the first millennium, bureaucratic computation had become the chore of other groups.

The easy version of the connection between the rise of the state and the development of mathematics (in Mesopotamia and elsewhere) is thus an illusion.

But politically, Wittvogel's book served his political patrons well for decades, just as did his often fanciful denunciations.