Any good and successful explanation has to be asymmetric, otherwise, it’s circular. In causal explanations, the explanatory asymmetry simply follows the direction of causation, i.e. we generally tend to think that causes explain their effects, and not the other way around. For example, if we say that the changes in the air temperature cause the mercury to expand and thus to climb up the glass column in a thermometer, it seems absurd to say that expanding of the mercury causes the changes in air temperature. If the changes in air temperature are true causes of the expending of the mercury in the thermometer, then this kind of asymmetry will have to be preserved across all the counterfactuals related to that explanation. That is why the counterfactual information and explanatory asymmetries are central in distinguishing good from bad explanations.

But in topological explanations in neuroscience it is not immediately obvious what can ground the explanatory asymmetry. This point is explicitly put forward in a recent paper by Craver and Povich (2016). They show that Lange’s (2012) cases of distinctively mathematical explanations fail to account for directionality of explanation if they don’t appeal to ontic facts. Their general argument is summarized as follows:
1. There are distinctively mathematical explanations of natural phenomena;
2. Mathematical explanations are directionless;
3. Explanations of natural phenomena are not directionless. (Craver and Povich 2017:

As one of the examples of distinctively mathematical explanation they discuss the famous 7 bridges of Koengisberg problem, and argue that any other kind of topological explanation should suffer from the same shortcomings, i.e. without appealing to ontic facts the topological explanation is directionless.

They formulate the argument as follows:
Why did Marta fail to walk a path through Königsberg in 1735, crossing each of its bridges exactly once (an Eulerian walk)?
1. Because, that year, Königsberg’s bridges formed a connected network with four nodes (landmasses); three nodes had three edges (bridges); one had five (EP).
2. But only networks that contain either zero or two nodes with an odd number of edges permit an Eulerian walk (MP) (489). (Craver and Povich 2017:3)
They then reverse the argument to show that Lange’s account is inadequate to account for the explanatory directionality:

“Why did either zero or two of Königsberg’s landmasses have an odd number of bridges in 1756?

Because Marta walked through town, hitting each bridge exactly once (EP), and

Only networks containing zero or two nodes with an odd degree contain an Eulerian path (MP).

As in the other examples, Königsberg’s layout is arguably better explained by the decisions of the Burgermeister than by Marta’s walk, yet facts about Königsberg’s layout follow reliably from descriptions of either.” (Craver and Povich 2017:4)

In his response, Marc Lange says that the reverse cases are not examples of explanations at all because the why question in those cases does not provide a context in which walking through all the bridges is a part of what it is for a certain arrangement to have a certain topological property P (the Eulerian walk).

I tend to agree with Lange, but that is not the main point of my paper, because all of the examples that Craver/Povich and Lange consider are more akin to toy examples.

I would like to focus on the actual examples from scientific practice, because I do think that the issue of directionality and ontic commitments in topological explanations is the key one for understanding this explanatory practice.

I show that there are two ways to think about non-causal directionality in describing counterfactual dependency relation and how they can ground the explanatory asymmetry, i.e. the “vertical” and the “horizontal”.

By “vertical”, I mean counterfactual dependency relation which describes dependency between variables at different levels or orders in mathematical hierarchy.

These are explanatory in virtue of constraining a range of variables in a counter-possible sense, i.e. had the constraining theorem been false it wouldn’t have had constrained the range of object level variables. Huneman states it explicitly: “In other words, were this mathematical proposition untrue, the system would not exhibit the property P” (Huneman 2017: 23). In this sense, the fact that a meta-variable or a higher order mathematical property holds entails that a mathematical property P obtains in the same class of variables or operations. In this sense, there is a direct entailment form that mathematical property to the explanandum P. (Huneman 2017: 24)

This gives a clue about the explanatoriness of mathematics in this kind of explanation, i.e. the truth of the meta-constraining variable does not depend on its specific value or the exact values of the lower level variables which it constrains. An example of this approach would be an explanation stability of an ecological community. Species and predation relations between them can be modelled as a graph which can have several salient network properties: clustering coefficient, connectance and the path length. By looking into the dependencies between the specific values of these graph properties one can predict various kinds of things (what would happen to the generalist species if some of the species went extinct). However, the combination of these network properties also entail a more general graph property such as being a “small-world” and that general property in turn entails various kinds of general properties, e.g. the stability or robustness that are entailed from various perturbations of the network though which it maintains the small-worldliness (Huneman 2017: 29).

On the other hand, by “horizontal” I mean the counterfactual dependency relations that are at same level or order in the mathematical hierarchy.

An example of “horizontal” counterfactual dependencies relations are the ones that hold between the topological variables such as the node’s weighted degree or the network communicability measure and the variables that describe system’s dynamics as a state space.
In the vertical case, the directionality seems to be straightforward to understand, it goes from higher order mathematical structure to lower level mathematical properties. In other words, the asymmetry follows the direction of derivation. This type of topological explanation, as Huneman readily admits does rely on the notion of modal strength a la Lange. However, it doesn’t require ontic details to be asymmetric.

I show that the modal strength in the “horizontal” type of topological explanation is more Woodwardian than Langean like.

This is because instead of describing the logical necessity underlying the mathematical facts in the explanans, in horizontal cases the modal strength comes from the counterfactuals that describe how would hypothetical changes in the values of topological variables affect the values of dynamical variables.

In “horizontal” cases such directionality may be conceived in terms of constraining dependency relations between topological structures and the network representation of the brain dynamics. In this sense, even though the topological and dynamic variables are at the same organizational level and the same order in the mathematical hierarchy, the constraining relations between them give the explanation its directionality.

To make my case I discuss the example of topological controllability which shows that some topological explanations have a structure of a counterfactual that describes horizontal dependency relations between topological properties and a network representation of the brain.

The horizontal cases can be sharply distinguished from the vertical cases because there the multitudes of ways in which the arrangements of topological properties can be used to describe the small-world topology, and the ways in which these arrangements provide different patterns of dependencies and constraints between topological structure and the dynamical features of the system (Hilgetag and Goulas 2017). Whereas vertical cases will be more akin to Lange’s toy examples, i.e. the system has certain property X given the mathematical fact Y.

The difference between the vertical and horizontal cases also have to do with different explananda.

For example the Watts and Strogatz model gives answers to coarse-grained and generic questions, e.g. why is the system stable or computationally efficient. But the modular and hierarchical instantiations of small-world topology provide answers to finer-grained questions.

For example, small-worldness that is rooted in hierarchically modular topology in the brain will be advantageous for the locally segregated processing in highly specialized functions (e.g. in visual motion detection) because the high clustering within the module will enable low wiring costs, and at the same time in such topology the short path lengths will more easily facilitate globally integrated processing of some of the more generic functions (e.g. working memory).

The vertical cases seem to be examples of distinctively mathematical explanations in Lange’s sense, and if the Craver/Povich arguments can be generalized, they might affect those too. But the horizontal cases are not like Langean cases, even though they are mathematical and non-causal. In horizontal cases the modal strength is more Woodwardian like.
Summary:
- Directionality in topological explanation can be conceived in terms of “vertical” and “horizontal” counterfactual dependency relations;
- I focused more on the horizontal way by discussing the topological controllability of the brain;
- Neither of these two ways appeals to ontic details in order to be asymmetric;
- The modal strength in the horizontal cases is more Woodwardian than Langean like and thus it is not susceptible to objections a la Craver/Povich.

List of references: